CS769 Advanced NLP

Language Modeling

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Slides adapted from Graham

https://junjiehu.github.io/cs769-fall25/

Goals for Today

- Problem definition
- **N-gram Language Model**
- **Neural Language Model**
- Evaluation

Are These Sentences OK?

- Jane went to the store.
- store to Jane went the.
- Jane went store.
- Jane goed to the store.
- The store went to Jane.
- The food truck went to Jane.

Engineering Solutions

- Jane went to the store.
- store to Jane went the.
- Jane went store.

Create a grammar of the language

- Jane goed to the store.

Consider morphology and exceptions

- The store went to Jane.

Semantic categories, preferences

- The food truck went to Jane.

And their exceptions

Quick Review of Probability

- Event space (e.g., $\mathscr{X}, \mathscr{Y}$)—in this class, usually discrete
- Random variables (e.g.,X, Y)
- Typical statement: "random variable X takes value $x \in \mathscr{X}$ with probability $P(X = x)$, or in shorthand, $P(x)$"
- Joint probability: $P(X = x, Y = y)$
- Conditional probability: $P(X = x \mid Y = y) = \dfrac{P(X = x, Y = y)}{P(Y = y)}$
- Bayes rule: $P(X, Y) = P(X \mid Y)P(Y) = P(Y \mid X)P(X)$
- Independent variables X, Y: $P(X, Y) = P(X)P(Y)$
- The difference between *true* and *estimated* probability distributions

Notation and Definitions

- $\mathscr{V}$ is a finite set of (discrete) symbols (e.g., words or characters); $V = |\mathscr{V}|$
- $\mathscr{V}*$ is the (infinite) set of sequences of symbols from $\mathscr{V}$
- In language modeling, we imagine a sequence of random variables $X = \langle x_1, x_2, \ldots, x_n \rangle$ that continues until $x_n =$ "[EOS]"
- $\mathscr{V}^+$ is the (infinite) set of sequences of $\mathscr{V}$ symbols, with the last token$x_n =$ "[EOS]"
- LM problem: Estimate the probability of a sequence $P(X)$, $X \in \mathscr{V}^+$

Language Modeling Problem

- Input: training data a sequence $X = \langle x_1, x_2, \ldots, x_n \rangle \in \mathscr{V}^+$
    - Sometimes it's useful to consider a collection of training sentences, each in $\mathscr{V}^+$, but it complicates notation.
- Output: $P : \mathscr{V}^+ \rightarrow \mathbb{R}$

$$P(X) = \prod_{i=1}^{I} P(\underbrace{x_i}_{\text{Next Word}} \mid \underbrace{x_1, \ldots, x_{i-1}}_{\text{Context}})$$

The big problem: How do we predict

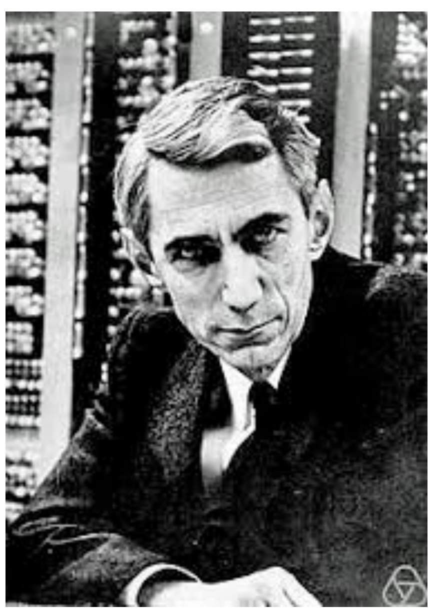
$$P(x_i \mid x_1, \ldots, x_{i-1})$$

?!?

Shannon's Language Model

(Shannon 1950)

- First seriously formulated study on LM
- Information theory: how much information does English as a language convey?



One method of calculating the entropy // is by a series of approximations 7? $F_1, F_2 >$ ''' , which successively take more and more of the statistics of the language into account and approach H as a limit. F_N may be called the N-gram entropy; it measures the amount of information or entropy due to statistics extending over N adjacent letters of text. F_N is given by[1]

$$FN = -\sum p(b_i, j) \log_2 p_{b_i}(j)$$

$$= -\sum_{i,j} p(b_i, j) \log_2 p(b_i, j) + \sum_i p(b_i) \log p(b_i) \qquad (1)$$

in which: bi is a block of N-1 letters [(N-1)-gram]

j is an arbitrary letter following b_i

$p(b_i, j)$ is the probability of the N-gram b_i, j

$p_{b_i}(j)$ is the conditional probability of letter j after the block b_i,

and is given by $p(b_i, j)/p(b_i)$.

The equation (1) can be interpreted as measuring the average uncertainty (conditional entropy) of the next letter /' when the preceding N-1 letters are known. As N is increased, F_N includes longer and longer range statistics and the entropy, H, is given by the limiting value of F_N as $N \to \infty$:

$$H = \lim_{N\to\infty} F_N . \qquad (2)$$

What Can we Do w/ LMs?

- Score sentences, e.g., $P(X =$ "Jane went to the store"$)$:

Jane went to the store . → high
store to Jane went the . → low

(same as calculating loss for training)

- Generate sentences:

while didn't choose end-of-sentence symbol, i.e., [EOS]:
  calculate probability $P($Next Word | Context$)$
  sample a new word from the probability distribution

N-gram Language Models

Review: Count-based Unigram Model

- **Independence assumption:** $P(x_i|x_1,\ldots,x_{i-1}) \approx P(x_i)$

- **Maximum-likelihood estimation (MLE): counting how likely each word appearing in a corpus**

$$P_{\mathrm{MLE}}(x_i) = \frac{c_{\mathrm{train}}(x_i)}{\sum_{\tilde{x}} c_{\mathrm{train}}(\tilde{x})}$$

- **Interpolation w/ UNK model:**

$$P(x_i) = (1-\lambda_{\mathrm{unk}}) * P_{\mathrm{MLE}}(x_i) + \lambda_{\mathrm{unk}} * P_{\mathrm{unk}}(x_i)$$

From Unigram to Bigram LM

- Next word prediction only depends on the previous word.

$$P(x_i|x_{i-n+1},\ldots,x_1) \approx P(x_i|x_{i-1})$$

- Given a training corpus of 3 sentences:

```
<s> I am Sam </s>
<s> Sam I am </s>
<s> I do not like green eggs and ham </s>
```

- Compute the bigram probability by counting (MLE):

$$P(\texttt{I}|\texttt{<s>}) = \frac{2}{3} = .67 \qquad P(\texttt{Sam}|\texttt{<s>}) = \frac{1}{3} = .33 \qquad P(\texttt{am}|\texttt{I}) = \frac{2}{3} = .67$$
$$P(\texttt{</s>}|\texttt{Sam}) = \frac{1}{2} = 0.5 \qquad P(\texttt{Sam}|\texttt{am}) = \frac{1}{2} = .5 \qquad P(\texttt{do}|\texttt{I}) = \frac{1}{3} = .33$$

- Probability of a sentence is just the product of all bigram probabilities in this sentence.

Higher-order *n*-gram Models

- Limit context length to *n*

$$P(x_i|x_{i-n+1},\ldots,x_1) \approx P(x_i|x_{i-n+1},...,x_{i-1})$$

- Maximum likelihood estimation: count, and divide

$$P_{ML}(x_i \mid x_{i-n+1},\ldots,x_{i-1}) := \frac{c(x_{i-n+1},\ldots,x_i)}{c(x_{i-n+1},\ldots,x_{i-1})}$$

$$\text{P(example | this is an)} = \frac{\text{c(this is an example)}}{\text{c(this is an)}}$$

- Add smoothing by linear interpolation with (n-1)-gram LM, to deal with zero counts:

$$\begin{aligned} P(x_i \mid x_{i-n+1},\ldots,x_{i-1}) =& \lambda P_{ML}(x_i \mid x_{i-n+1},\ldots,x_{i-1}) \\ &+ (1-\lambda)P(x_i \mid x_{1-n+2},\ldots,x_{i-1}) \end{aligned}$$

Smoothing for words in unknown context

- Add-one smoothing, i.e., every word adds 1 count.

$$P(x_i) = \frac{\text{count}(x_i)}{N}$$

$$P(x_i) = \frac{\text{count}(x_i) + 1}{N + V}$$

- Add smoothing by linear interpolation with (n-1)-gram LM, to deal with zero counts:

$$\begin{aligned} P(x_i \mid x_{i-n+1}, \ldots, x_{i-1}) = &\lambda P_{ML}(x_i \mid x_{i-n+1}, \ldots, x_{i-1}) \\ &+ (1-\lambda) P(x_i \mid x_{1-n+2}, \ldots, x_{i-1}) \end{aligned}$$

More Smoothing Methods
(e.g. Goodman 1998)

- **Additive/Dirichlet:**

fallback distribution

$$P(x_i \mid x_{i-n+1}, \ldots, x_{i-1}) := \frac{c(x_{i-n+1}, \ldots, x_i) + \boxed{\alpha}\boxed{P(x_i \mid x_{i-n+2}, \ldots, x_{i-1})}}{c(x_{i-n+1}, \ldots, x_{i-1}) + \boxed{\alpha}}$$

interpolation hyperparameter

- **Discounting:**

discount hyperparameter

$$P(x_i|x_{i-n+1}, \ldots, x_{i-1}) := \frac{c(x_{i-n+1}, \ldots, x_i) - \boxed{d} + \boxed{\alpha} P(x_i|x_{i-n+2}, \ldots, x_{i-1})}{c(x_{i-n+1}, \ldots, x_{i-1})}$$

interpolation calculated by sum of discounts $\boxed{\alpha} = \sum_{\{\tilde{x}; c(x_{i-n+1}, \ldots, \tilde{x}) > 0\}} d$

- **Kneser-Ney:** discounting w/ modification of the lower-order distribution

Goodman. An Empirical Study of Smoothing Techniques for Language Modeling. 1998.

Problems and Solutions?

- Cannot share strength among **similar words**

she bought a car	she bought a bicycle
she purchased a car	she purchased a bicycle

→ solution: class based language models

- Cannot condition on context with **intervening words**

Dr. Jane Smith   Dr. Gertrude Smith

→ solution: skip-gram language models

- Cannot handle **long-distance dependencies**

for tennis class he wanted to buy his own racquet

for programming class he wanted to buy his own computer

→ solution: cache, trigger, topic, syntactic models, etc.

Neural Language Model

—Beyond Linear Models

Linear Models can't Learn Feature Combinations

students take tests→ **high** teachers take tests → **low**

students write tests → **low** teachers write tests → **high**

- These can't be expressed by linear features
- What can we do?
  - Remember combinations as features (individual scores for "students take", "teachers write")
→ Feature space explosion!
  - Neural networks!

Neural Language Models

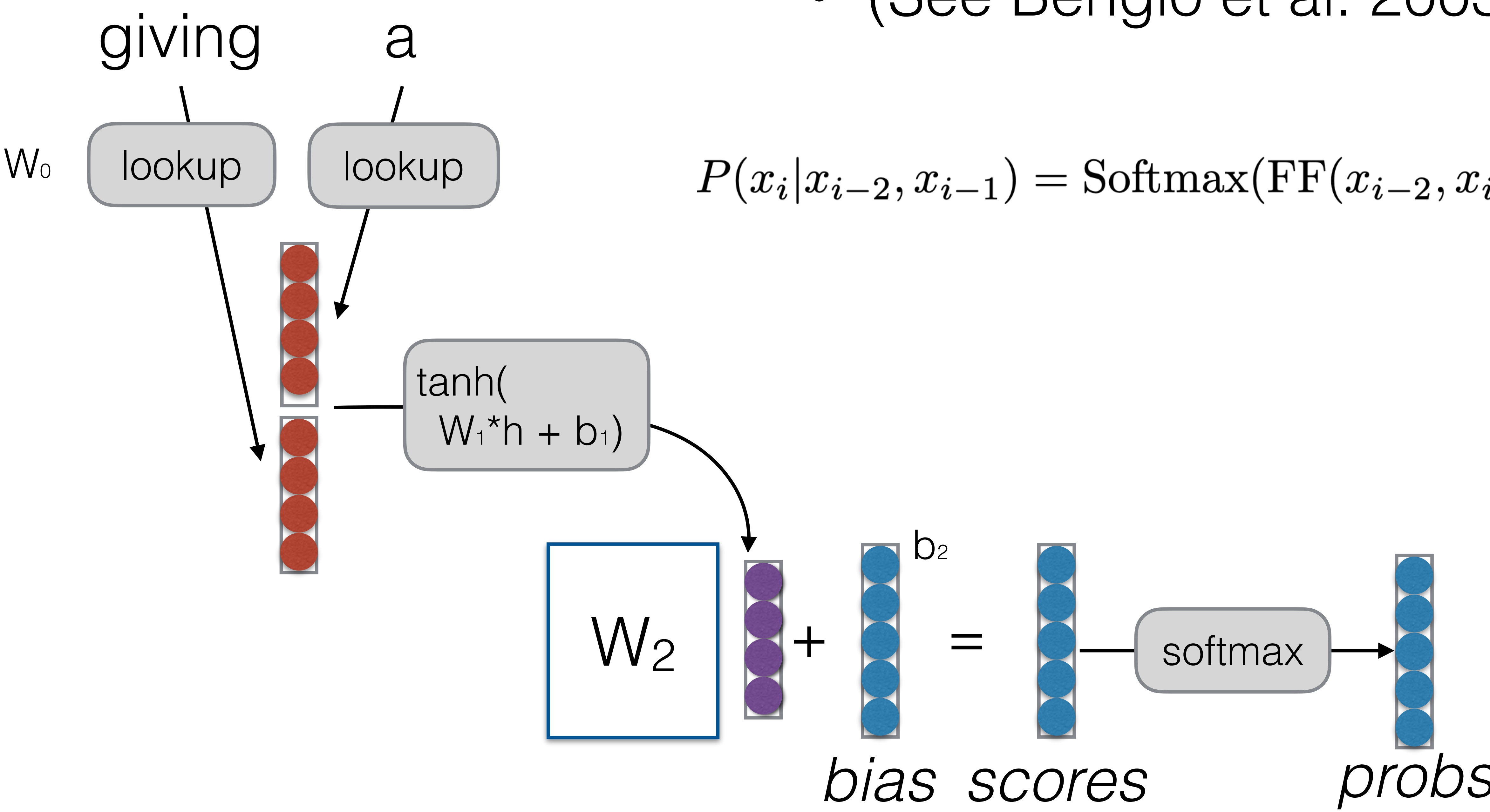
- Convert the word prediction problem to discriminative text classification
- The input is the n-1 previous words (context)
- The output is a word in the vocabulary (V-class classification)

$$P(x_i|x_{i-n+1},\ldots,x_{i-1}) = \mathrm{Softmax}(\mathrm{NeuralNet}(x_{i-n+1},\ldots,x_{i-1}))$$

Feed-forward Neural Language Models

- (See Bengio et al. 2003)

$$P(x_i|x_{i-2}, x_{i-1}) = \text{Softmax}(\text{FF}(x_{i-2}, x_{i-1}))$$



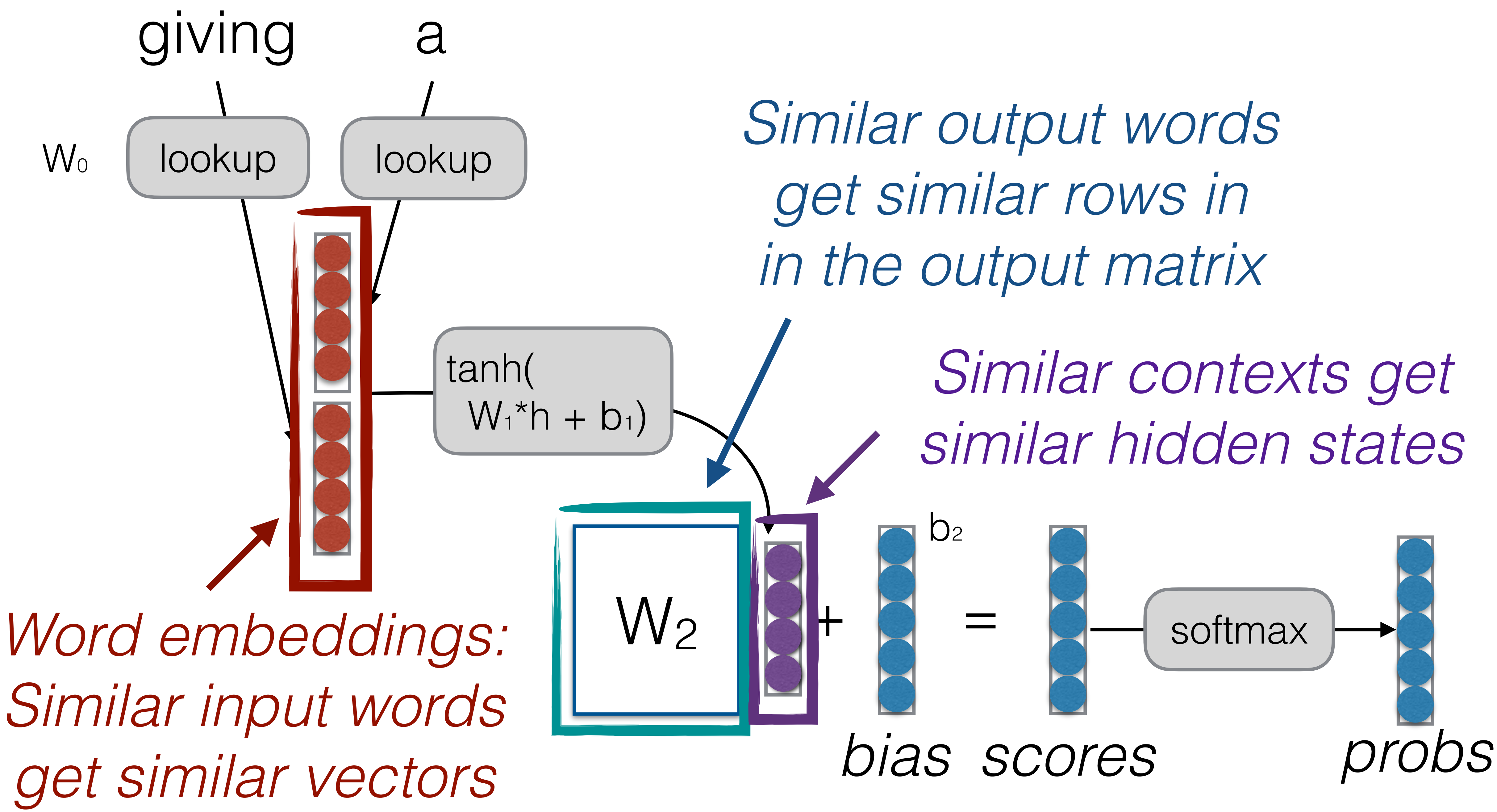
Example of Combination Features

- Word embeddings capture features of words
  - e.g. feature 1 indicates verbs, feature 2 indicates determiners
- A row in the weight matrix (together with the bias) can capture particular *combinations* of these features
  - e.g. the 34th row in the weight matrix looks at feature 1 in the second-to-previous word, and feature 2 in the previous word

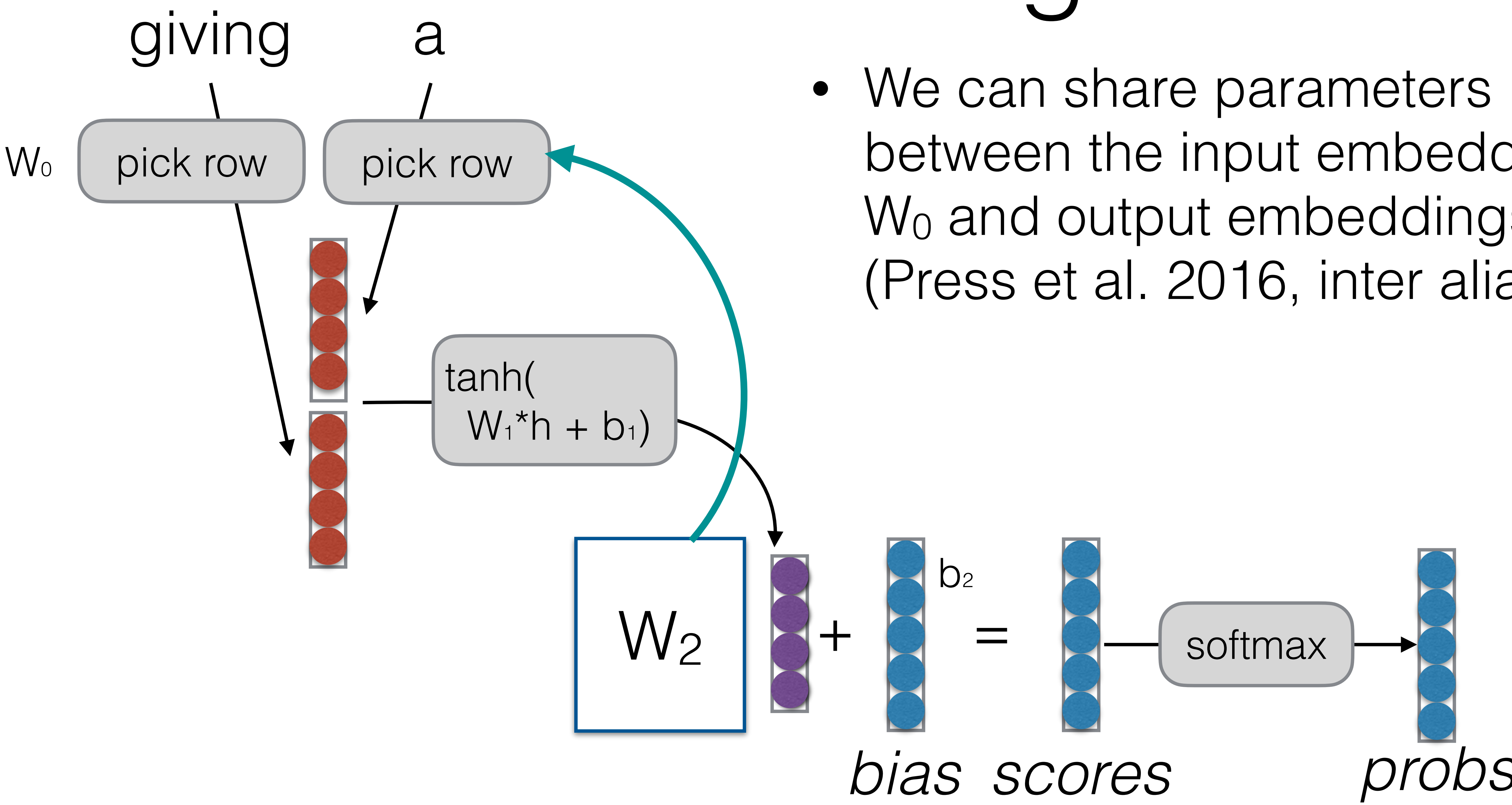
$$\begin{array}{rc} \text{giving} & \begin{bmatrix}1.2\\-0.1\\0.7\\-2.1\\0.5\end{bmatrix} \\ \text{a} & \begin{bmatrix}-0.3\\2.0\\0.6\\-0.8\\-0.4\end{bmatrix}\end{array} * \begin{array}{c}\boldsymbol{w}_{34}\\ \begin{bmatrix}1.5\\0\\0\\0\\0\\0\\1.3\\0\\0\\0\end{bmatrix}\end{array} + \begin{array}{c}b_{34}\\ -2\end{array} =$$

positive number if the previous word is a determiner and second-to-previous word is a verb

Where is Strength Shared?



Tying Input/Output Embeddings



- We can share parameters between the input embeddings W_0 and output embeddings W_2 (Press et al. 2016, inter alia)

Want to try? Delete the input embeddings W_0, and instead pick a row from the output matrix W_2.

What Problems are Handled?

- Cannot share strength among **similar words**

she bought a car	she bought a bicycle
she purchased a car	she purchased a bicycle

→ solved, and similar contexts as well! 😀

- Cannot condition on context with **intervening words**

Dr. Jane Smith   Dr. Gertrude Smith

→ solved! 😀

- Cannot handle **long-distance dependencies**

for tennis class he wanted to buy his own racquet

for programming class he wanted to buy his own computer

→ not solved yet 😞

Many Other Potential Designs!

- Neural networks allow design of arbitrarily complex functions!

- In future classes, we can replace Feedforward models by more powerful sequential NNs:

  - **Recurrent neural network LMs**

  - **Transformer LMs**

When to Use n-gram Models?

- Neural language models (next) achieve better performance, but
- n-gram models are extremely fast to estimate/apply
- n-gram models can be better at modeling low-frequency phenomena
- **Toolkit:** kenlm

https://github.com/kpu/kenlm

Comparison Between N-gram LM and Neural LMs

N-gram LM
(Nonparametric Modeling)

- **Count**: Build a table of all possible prefix contexts and their counts from the training corpus $\mathcal{D}$

$O(|\mathcal{D}|)$

Corpus size

Prefix Context	Next Word	Count
$x_{i-n+1}, \cdots, x_{i-2}, x_{i-1}$	x_i	$c(x_{i-n+1}, \cdots, x_i)$

N-gram LM

(Nonparametric Modeling)

- **Retrieve**: How to compute $P(x_i|x_{i-n+1},\cdots,x_{i-1})$?
  - Retrieve all rows that match the prefix
  - Divide the count of $(x_{i-n+1},\cdots,x_{i-1},x_i)$ by the sum of counts of all retrieved rows

$$P(x_i|x_{i-n+1},\cdots,x_{i-1}) \approx \frac{c(x_{i-n+1},\cdots,x_{i-1},x_i))}{\sum_{x'\in\mathcal{V}} c(x_{i-n+1},\cdots,x_{i-1},x')}$$

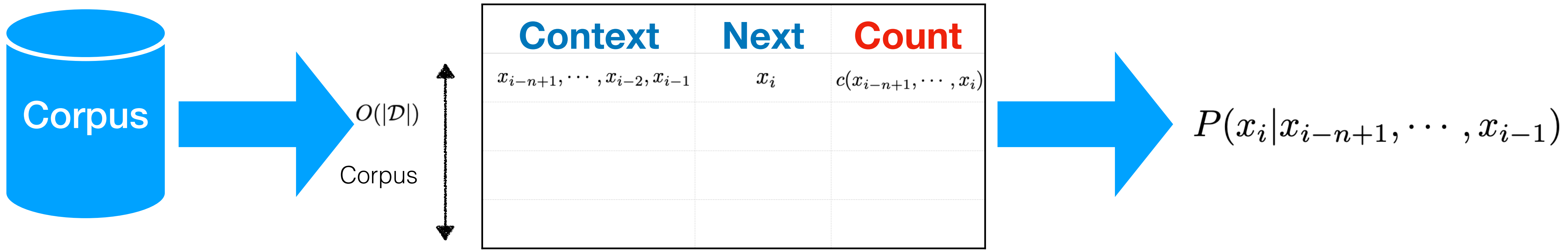
$O(|\mathcal{D}|)$

Corpus

Prefix Context	Next Word	Count
$x_{i-n+1},\cdots,x_{i-2},x_{i-1}$	x_i	$c(x_{i-n+1},\cdots,x_i)$

Count-based Language Modeling

- Count & Retrieve!



Neural Language Model

(Bengio et al. 2003)

- **Non-parametric to parametric**: We replace a count table with a neural network to approximate the next-word prediction probability:

$$P_\theta(x_i | x_{i-n+1}, \cdots, x_{i-1}) = \mathrm{Softmax}(\mathrm{NeuralNet}(x_{i-n+1}, \cdots, x_{i-1}))$$

θ is the LM parameter

- **Training**: Use SGD to optimize the network by minimizing the **negative log-likelihood** of next word prediction

$$\mathcal{L}_{\mathrm{nll}}(\theta) = -\mathbb{E}_{x_i | x_{<i}} \log P_\theta(x_i | x_{i-n+1}, \cdots, x_{i-1})$$

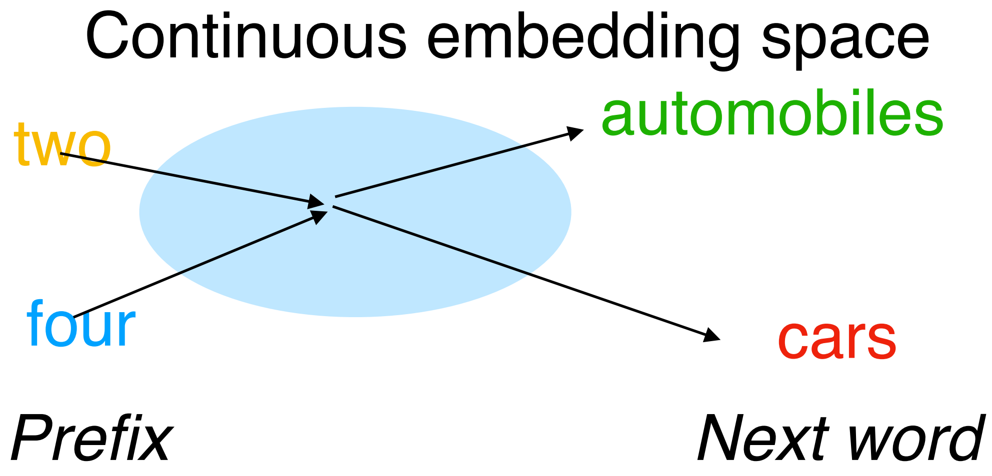
Sample a pair of (prefix context, next word) from a large corpus

Neural LM v.s. Count LM

- A neural LM *learns to* ***count***, but also *learns to* ***compress***
  - Learning is ***not*** simply counting
  - Neural nets ***memorize*** texts but also ***generalize***
  - Similar text inputs are clustered to save ***no. of bits***
  - Similarity is implicitly imposed by similarity between ***prefix context*** and ***next word***

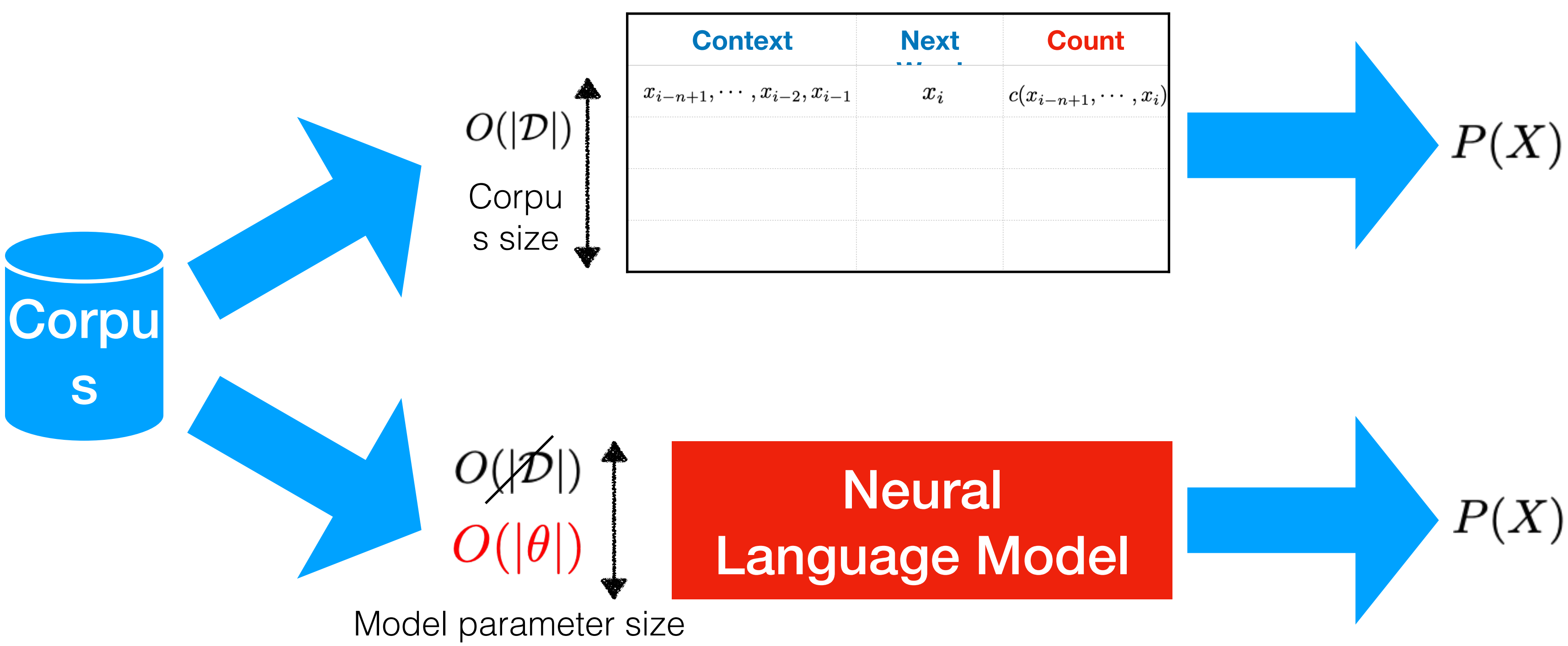
Example:

- Training examples
  - Peter and Mary have two cars.
  - There are four cars on the road.
  - The parking lots have four automobiles.
- Q: How likely is “automobiles” followed by “two”?



Neural LM learns to count & compress

- Neural LM generalizes by compressing the no. of rows



Non-parametric

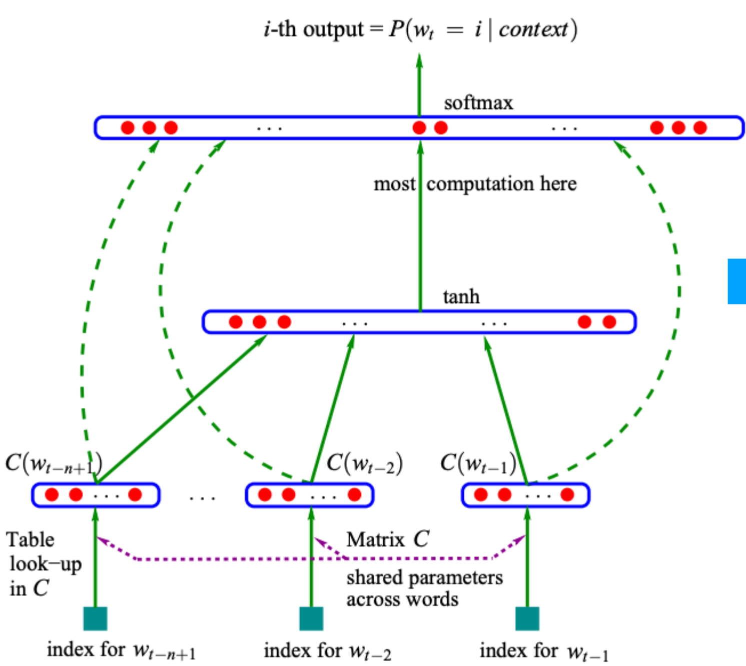
- Memorize low-frequency pairs of (prefix, next word)
- Cannot generalize to unseen pairs
- Interpretable & easy for incremental updates (insert/delete/replace)

Parametric

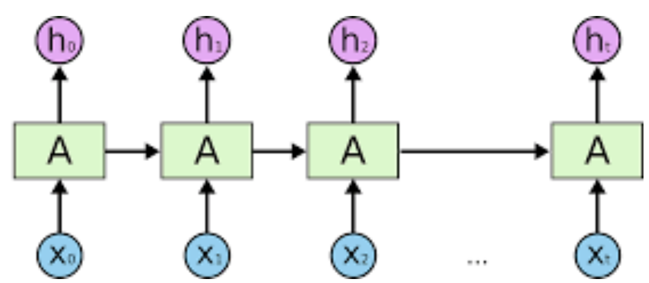
- Learning from data-driven training
- Good generalization to unseen pairs
- Black-box & expensive for parameter updates with forgetting issues

What makes LLM so powerful?

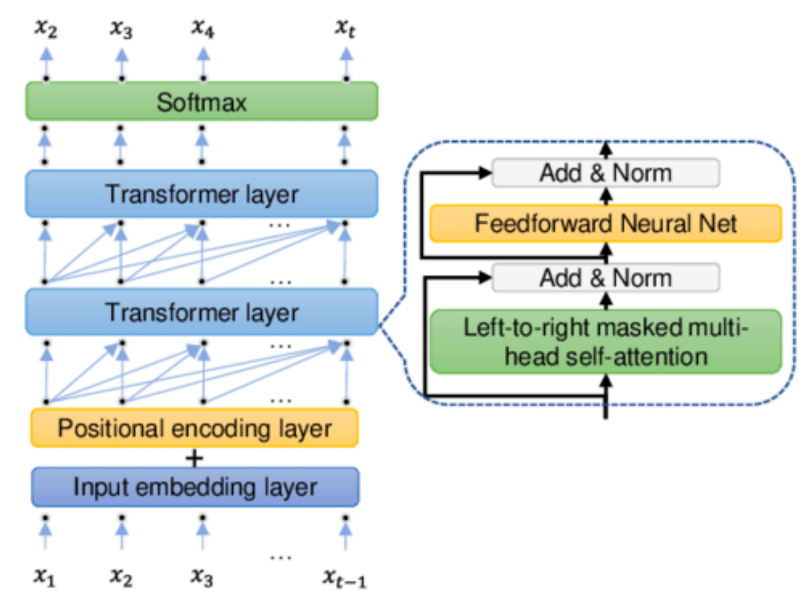
- **Advances from neural network architecture:**
  - Scale up the size of model parameters to 100B+



MLP



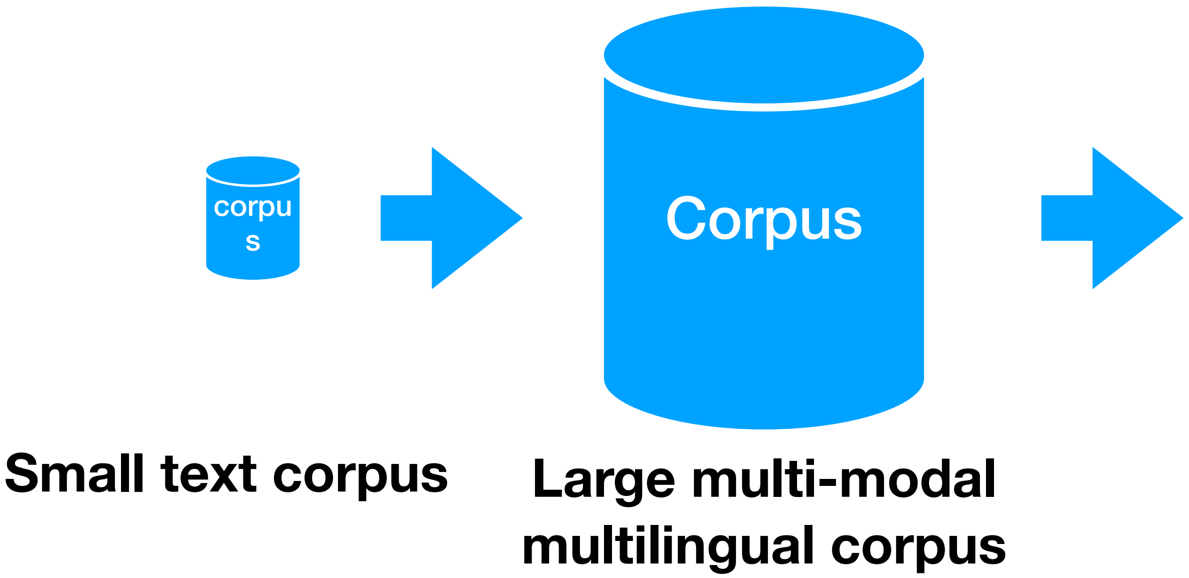
RNN

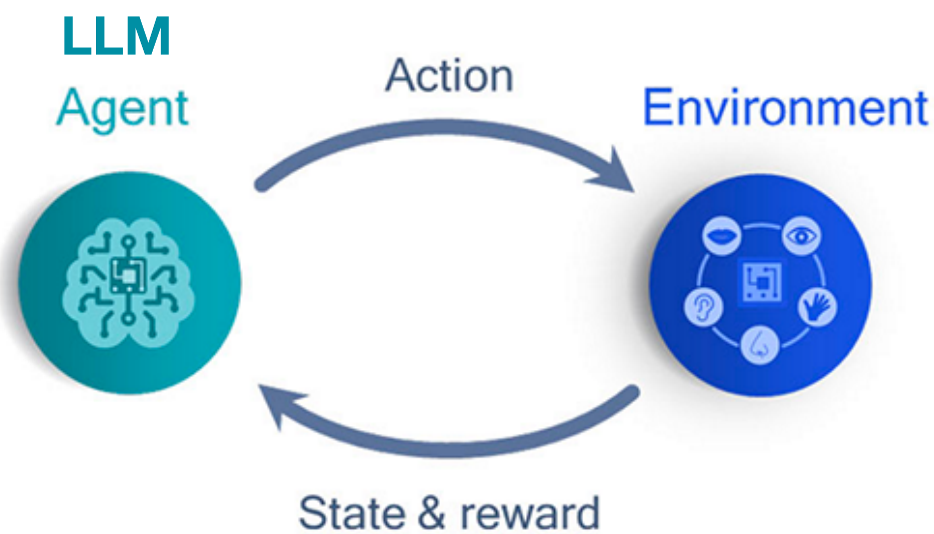


Transformer

What makes LLM so powerful?

- **Pre-training** on massive raw **texts** (and even **images**) in **100+ languages**
  - BERT, GPT-4, PaLM, T5, LLaMA, …
- **Fine-tuning** on language **instructions** with **supervised learning** or **RL with human feedback**
  - ChatGPT, Bard, FlanT5, Alpaca, …





LM Evaluation

Evaluation of LMs

- **Log-likelihood:**

$$LL(\mathcal{D}_{\text{test}}) = \sum_{X \in \mathcal{D}_{\text{test}}} \log P(X)$$

- **Per-word Log Likelihood:**

$$WLL(\mathcal{D}_{\text{test}}) = \frac{1}{\sum_{X \in \mathcal{D}_{\text{test}}} |X|} \sum_{X \in \mathcal{D}_{\text{test}}} \log P(X)$$

- **Per-word (Cross) Entropy:**

$$H(\mathcal{D}_{\text{test}}) = \frac{1}{\sum_{X \in \mathcal{D}_{\text{test}}} |X|} \sum_{X \in \mathcal{D}_{\text{test}}} -\log_2 P(X)$$

- **Perplexity:**

$$ppl(\mathcal{D}_{\text{test}}) = 2^{H(\mathcal{D}_{\text{test}})} = e^{-WLL(\mathcal{D}_{\text{test}})}$$

Unknown Words

- Necessity for UNK words
    - We won’t have all the words in the world in training data
    - Larger vocabularies require more memory and computation time
- Common ways:
    - Limit vocabulary by frequency threshold (usually UNK <= 1) or rank threshold
    - Model characters or subwords

Evaluation and Vocabulary

- **Important:** the vocabulary must be the same over models you compare
- Or more accurately, all models must be able to generate the test set (it’s OK if they can generate *more* than the test set, but not less)
    - e.g. Comparing a character-based model to a word-based model is fair, but not vice-versa

LM Problem Definition

Count-based LMs

Evaluating LMs

Log-linear LMs

Neural Net Basics

Feed-forward NN LMs

Questions?